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Identification of peaks in multidimensional coincidence γ-ray spectra

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Abstract

In the paper a new algorithm to find peaks in two, three and multidimensional spectra, measured in large multidetector γ-ray arrays, is derived. Given the dimension *m*, the algorithm is selective to *m*-fold coincidence peaks. It is insensitive to intersections of lower-fold coincidences, hereinafter called ridges.

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1. Introduction

The event data obtained with large γ-ray detector arrays such as EUROGAM [1], GAMMASPHERE [2], GASP [3] can be used to generate two, three, four or in general *m*-fold coincidence spectra. Progress in the understanding of nuclear structure depends on the ability to analyze these multidimensional spectra correctly. Obviously, the lack of methods for analyzing such data sets can create a major obstacle to extracting interesting physical information. The information contained in the many-fold coincidence spectra is overwhelming and so extracting all the detailed information contained in multidimensional peaks proves difficult. The evaluation of multidimensional peaks starts with determining their position in a multidimensional space. Subsequently, the peak-searching procedure must recognize the shape of the object found, and decide whether it is multidimensional Gaussian or not. Afterward, the positions of identified peaks can be fed as initial estimates into the peak-fitting procedure for thorough analysis.

At present, there exist several algorithms for automatic identification of peaks in one-dimensional spectra. One such well-known peak-searching algorithm is Mariscotti's method [4], which is based on the evaluation of smoothed second differences. A modification of this method is presented in Ref. [5]. The basis of the methods presented in Refs. [6–8] is the smoothed first derivative which changes sign in the vicinity of the

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peak. A different approach based on Markov chains is used in the peak-searching method published in Ref. [9].

Application of any of these methods cannot be directly extended to searching for peaks in a multidimensional space. In a simplified way one can think of a one-dimensional spectrum as a composition of background, peaks and statistical disturbances (noise). However, in multidimensional γ-ray spectra, things are completely different. For instance two-dimensional γ-ray spectra, contain continuous background, one-fold coincidences (ridges as a result of peak–background coincidences) in both directions and two-fold coincidence γ-ray peaks. We are only interested in, and looking for, two-fold coincidence peaks. The proposed algorithm must be insensitive to the intersections of one-fold coincidence ridges that, at first glance, look like two-dimensional peaks. The method must be able to distinguish between the intersection of two ridges and a two-fold coincidence peak. The situation is further complicated by the fact that two-dimensional peaks are usually located at the points of crossing ridges.

Analogously three-dimensional spectra contain continuous background, one-fold and two-fold coincidences (ridges), in all three directions, as well as three-fold coincidence peaks. Again, we are interested only in the identification of three-fold coincidence peaks. The algorithm should ignore the intersections of either one-fold or two-fold coincidence ridges. In general, in m-dimensional γ-ray spectra the algorithm should identify only m-fold coincidence peaks.

The aim of the paper is to propose an algorithm for the identification of $2, 3, \ldots, m$-fold coincidence peaks in $2, 3, \ldots, m$-dimensional γ-ray spectra.

2. Peak searching algorithm in two-dimensional spectra

We suppose that the number of counts in a two-dimensional γ-ray spectrum in the channels x, y can be approximated by

$$N(x,y) = G_1(x,y) + G_2(x) + G_3(y) + B + Cx + Dy. \tag{1}$$

The two-dimensional Gaussian is defined as

$$G_1(x,y) = A_1 \exp\left\{ -\frac{1}{2(1-\rho^2)}\left[\frac{(x-x_1)^2}{\sigma_{x_1}^2} - \frac{2\rho(x-x_1)(y-y_1)}{\sigma_{x_1}\sigma_{y_1}} + \frac{(y-y_1)^2}{\sigma_{y_1}^2}\right]\right\} \tag{2}$$

where x_1, y_1 give its position, σ_{x_1} and σ_{y_1} are standard deviations of normal distributions in the directions x and y respectively, and ρ is a correlation coefficient. Analogously one-dimensional ridges parallel with axes y and x can be described by

$$G_2(x) = A_2 \exp\left\{ -\frac{(x-x_2)^2}{2\sigma_{x_2}^2}\right\} \tag{3}$$

and

$$G_3(y) = A_3 \exp\left\{ -\frac{(y-y_3)^2}{2\sigma_{y_3}^2}\right\} \tag{4}$$

respectively. For a short interval we assume that the background in Eq. (1) can be approximated by the linear function

$$B + Cx + Dy. \tag{5}$$

Obviously the mixed partial second derivative of Eq. (1) in both dimensions $\partial^4 N(x,y)/\partial x^2 \partial y^2$ does not depend on $G_2(x), G_3(y)$ and the linear background.

Due to the discrete nature of channel counts, we shall replace the mixed partial second derivatives by the mixed partial second differences in both dimensions, thus

$$S(i,j) = N(i+1,j+1) - 2N(i,j+1) + N(i-1,j+1) - 2N(i+1,j) + 4N(i,j) - 2N(i-1,j) + N(i+1,j-1) - 2N(i,j-1) + N(i-1,j-1). \quad (6)$$

In order to eliminate statistical fluctuations we shall employ smoothed mixed partial second differences in both dimensions.

The smoothing is achieved by summing $S(i,j)$ in a given window

$$S_w(i,j) = \sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} S(i_1,j_1) \quad (7)$$

where the window is $w = 2r + 1$. In accordance with Ref. [4] the repetitive application of Eq. (3) yields the general smoothed mixed partial second differences in both dimensions, $S_{z,w}(i,j)$, where z is the number of repetitions. For the sake of simplicity, from now on in the paper we shall call the difference spectrum $S_{z,w}(i,j)$ the two-parameter smoothed second differences or two-parameter SSD.

The contents of the channel i,j of this difference spectrum can then be expressed as

$$S_{z,w}(i,j) = \sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} C_{z,w}(i_1 - i, j_1 - j) N(i_1,j_1) \quad (8)$$

where $r = zm + 1$, $m = (w-1)/2$. Eqs. (6) and (7) imply that the elements of the filter matrix $C_{z,w}(i,j)$ can be factorized, i.e.

$$C_{z,w}(i,j) = C_{z,w}(i) C_{z,w}(j). \quad (9)$$

The vector elements $C_{z,w}(i)$, $i \in \langle -r, r \rangle$, can be calculated using the recursion formula

$$C_{z,w}(i) = \sum_{i_1=i-r}^{i+r} C_{z-1,w}(i_1) \quad (10)$$

where $i \in \langle -zr-1, zr+1 \rangle$ and

$$C_{0,w} = \begin{cases} -2 & \text{if } i = 0 \\ 1 & \text{if } |i| = 1 \\ 0 & \text{otherwise.} \end{cases}$$

In analogy with the algorithm in Ref. [4] and in view of Eq. (9), we define the standard deviation of a two-parameter SSD as

$$F_{z,w}(i,j) = \sqrt{\sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} C_{z,w}^2(i_1 - i) C_{z,w}^2(j_1 - j) N(i_1,j_1)}. \quad (11)$$

Another way to compute the coefficients of the filter vector C is used in Ref. [5]. The vector C is defined as the second derivative of the Gaussian

$$C_\sigma(i) = \frac{\mathrm{d}^2}{\mathrm{d}x^2} \exp\left(-\frac{x^2}{2\sigma^2} \right)\Bigg|_{x=i} \quad (12)$$

where σ is the standard deviation of the searched peaks. The replacement of the filter coefficients $C(i)$ from Eq. (12) to Eqs. (8), (9) and (11) is straightforward. The choice of method employed in a peak-searching procedure is optional and depends on the application. While in the first case the free parameters are z, w, in the second method the only free parameter is σ. In general, independently of the calculation method of the SSD filter, relations (8) and (11) can be expressed as

$$S(i,j) = \sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} C(i_1 - 1)C(j_1 - j)N(i_1,j_1) \tag{13}$$

$$F(i,j) = \sqrt{\sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} C^2(i_1 - i)C^2(j_1 - j)N(i_1,j_1)}. \tag{14}$$

The two-parameter SSD spectrum calculated using the algorithm presented above is insensitive to the intersection of two ridges representing coincidence peak–background. For illustration, using Eqs. (1)–(5) we have generated a (synthetic) spectrum (Fig. 1) with two two-dimensional Gaussians

$$G_{1A}(A_{1A} = 100, \quad x_{1A} = 60, \quad y_{1A} = 8)$$

$$G_{1B}(A_{1B} = 50, \quad x_{1B} = 180, \quad y_{1B} = 200) \tag{15}$$

two ridges parallel with the y axis

$$G_{2A}(A_{2A} = 200, \quad x_{2A} = 60)$$

$$G_{2B}(A_{2B} = 180, \quad x_{2B} = 180) \tag{16}$$

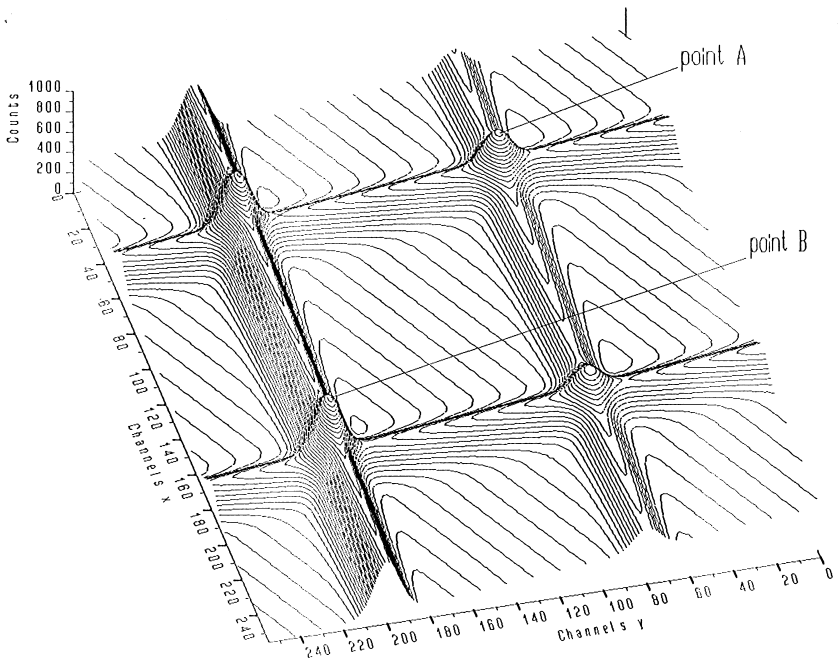

Fig. 1. Synthetic spectrum with two two-dimensional Gaussians located at points A and B.

two ridges parallel with the x axis

$$G_{3A}(A_{3A} = 200, \quad x_{3A} = 80)$$

$$G_{3B}(A_{3B} = 500, \quad x_{3B} = 200) \tag{17}$$

and linear background

$$800 - 1.176x - 1.96y.$$

The parameters σ were set to 6 and $\rho = 0$.

At a first glance it is very difficult to estimate the positions of two-dimensional Gaussians. Their positions are denoted in Fig. 1 by the points A, B. The difference spectrum calculated according to Eqs. (8)–(10) with $w = 7$, $z = 7$ is shown in Fig. 2.

Apparently the difference spectrum reflects only the presence of two-dimensional peaks G_{1A}, G_{1B}. It ignores linear background, ridges $G_{2A}, G_{2B}, G_{3A}, G_{3B}$ and their intersections.

Now we shall have a closer look at the shape of the difference spectrum of a two-dimensional peak (Fig. 3).

We can observe that in slices A, B, C, D the courses of the curves correspond to the smoothed second difference spectrum of a one-dimensional Gaussian. In slices E, F they correspond to negation of the smoothed second difference spectrum of a one-dimensional Gaussian. The difference spectrum and its standard deviation for one-dimensional Gaussian are given in Fig. 4.

According to Fig. 4, to decide whether an object found is a Gaussian peak or not, we have to define the points i_1–i_6. Subsequently we can apply evaluation criteria for a one-dimensional Gaussian described in detail in Ref. [4]. We repeat this procedure for each dimension (in our example slices E, F).

When looking at Figs. 2 and 3 more thoroughly, we can observe that one two-dimensional peak gives rise to five local maxima in the difference spectrum. The position of one of the local maxima located in the middle

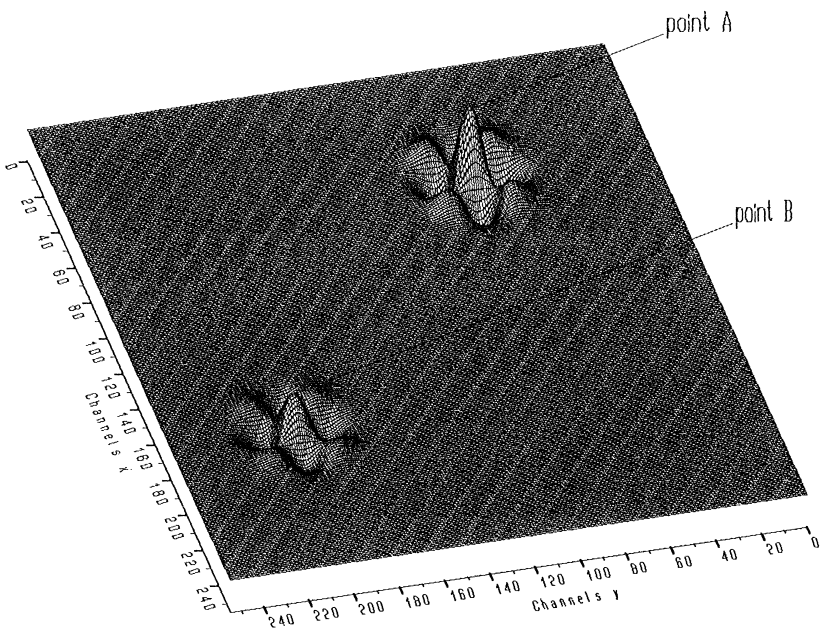

Fig. 2. The difference spectrum for the spectrum from Fig. 1.

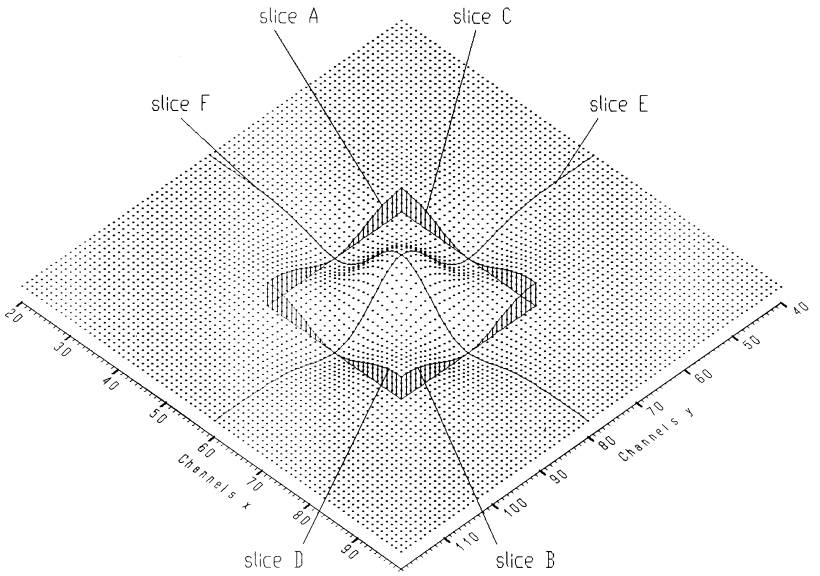

Fig. 3. The shape of the difference spectrum of the two-dimensional Gaussian.

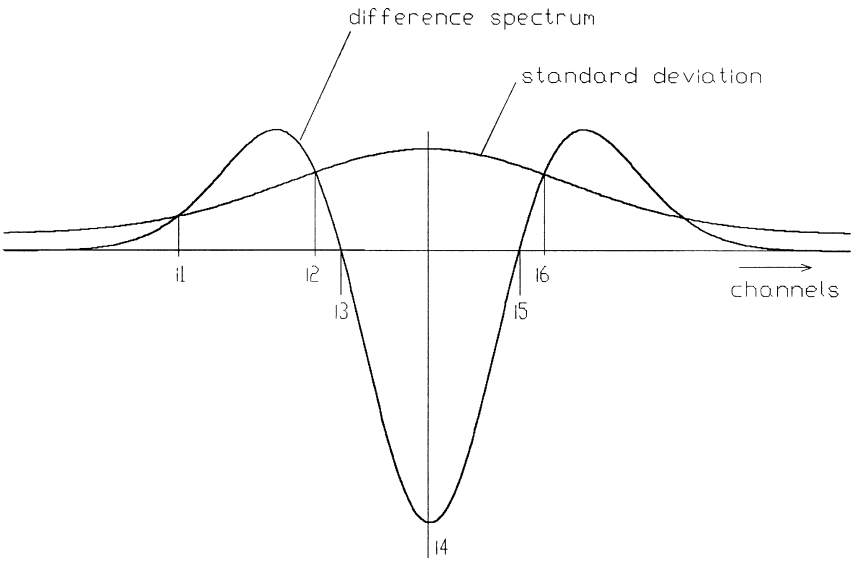

Fig. 4. The difference spectrum and its standard deviation for one-dimensional Gaussian.

of the surrounding ones corresponds to the position of the two-dimensional peak. Let us call this local maximum true and the remaining four local maxima false (Fig. 5).

Once we have calculated two-parameter SSD (difference spectrum), the problem is how to distinguish between true and false maxima. False maxima should be ignored by the searching algorithm. In simple instances, when we have just a few peaks in the spectrum, it may seem trivial. It can be solved by finding

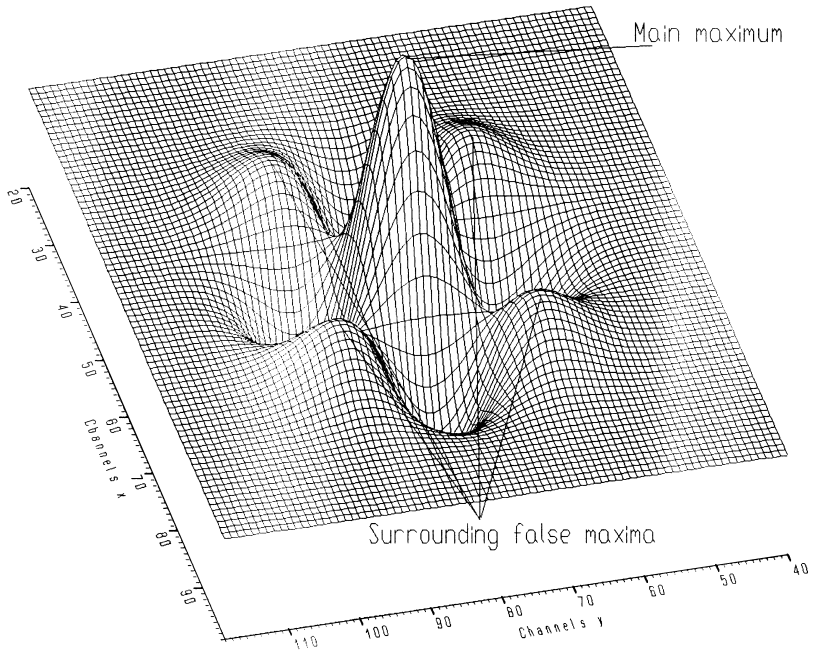

Fig. 5. True and false local maxima for the two-dimensional peak.

groups of corresponding five maxima. Such an example can be illustrated by the spectrum with one peak from an experiment of positron annihilation (Fig. 6) [10].

Its two-parameter SSD is shown in Fig. 7.

However, when a large number of peaks has to be evaluated in a spectrum, as is usually the case for γ-ray spectra, this task is very difficult. Figs. 8 and 9 give an example of such a two-dimensional γ-ray spectrum and its two-parameter SSD, respectively.

From Fig. 9 it is obvious that it is practically impossible to find groups of corresponding five local maxima. The true and false local maxima of two-parameter SSD of neighboring peaks in an original spectrum can either be mixed or may coincide. Therefore it is necessary to suppress in a way the false maxima in the two-parameter SSD.

In keeping with Eqs. (7), (8) and (13) we shall define spectra of the one-parameter smoothed second derivatives (one-parameter SSD) in both x and y independent variables, i.e.

$$X(i,j) = \sum_{i_1=i-r}^{i+r} C(i_1 - i)N(i_1,j) \tag{18}$$

and

$$Y(i,j) = \sum_{j_1=j-r}^{j+r} C(j_1 - j)N(i,j_1) \tag{19}$$

where the coefficients of the filter C are defined either according to Eq. (10) or Eq. (12).

As a step toward understanding the difference among filters, we provide an example in Table 1 of the filter $C_s(i,j)$ for $z = w = 0$ according to Eq. (10) for two-parameter SSD as well as examples of filters $C_x(i)$, $C_y(j)$ for one-parameter SSDs in a two-dimensional space.

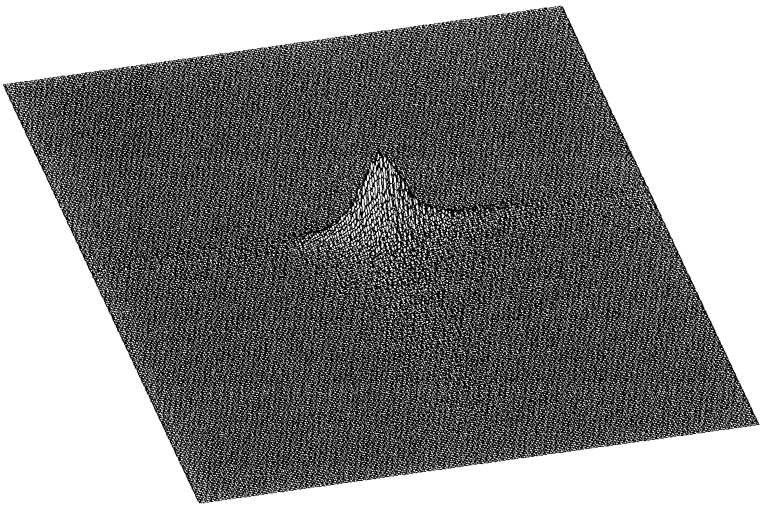

Fig. 6. Spectrum with one peak from experiment of positron annihilation.

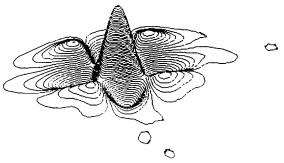

Fig. 7. Two-parameter SSD spectrum from Fig. 6.

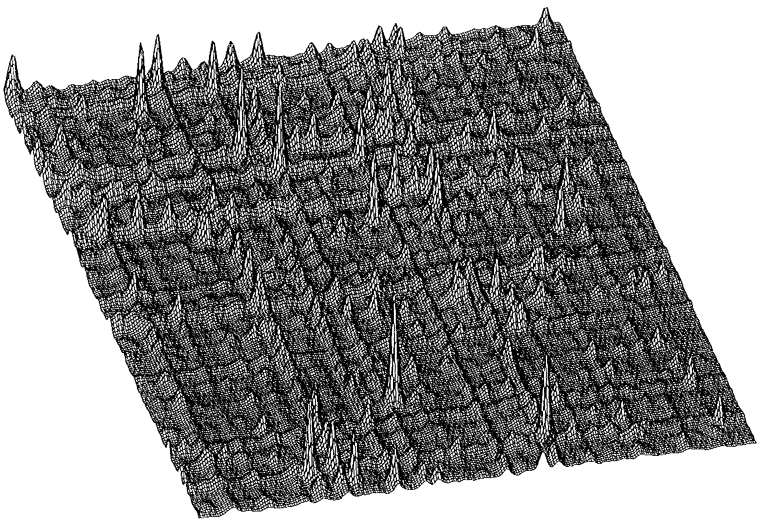

Fig. 8. Two-dimensional γ-ray spectrum.

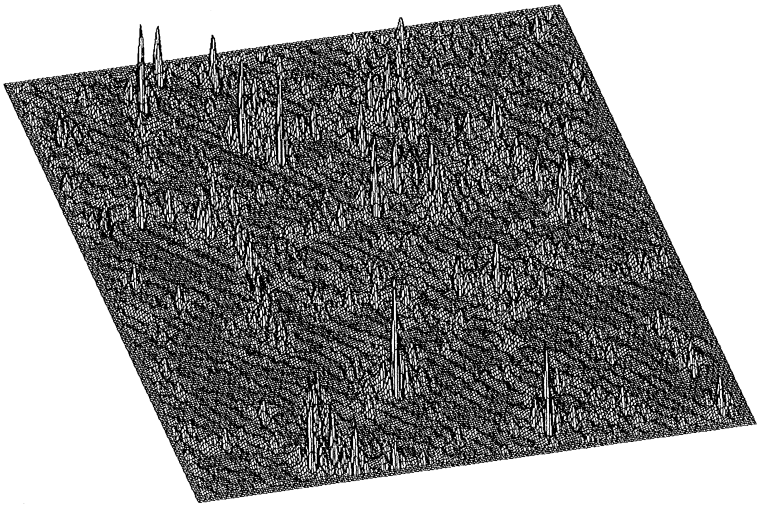

Fig. 9. Two-parameter SSD spectrum from Fig. 8.

Table 1
Examples of the filters $C_s(i,j)$ for $z = w = 0$ according to Eq. (10) for two-parameter SSD and $C_x(i), C_y(j)$ for one-parameter SSDs in two-dimensional space

	$C_s(i,j)$			$C_x(i,j)$			$C_y(i,j)$		
$j_1 \quad i_1$	$i-1$	i	$i+1$	$i-1$	i	$i+1$	$i-1$	i	$i+1$
$j-1$	1	−2	1	1	−2	1	1	1	1
j	−2	4	−2	1	−2	1	−2	−2	−2
$j+1$	1	−2	1	1	−2	1	1	1	1

From the filters defined in this way we can observe that, at the expected position of a two-dimensional peak $i_1 = i$, $j_1 = j$, we have

$$C_s(i,j) > 0, \quad C_x(i) < 0, \quad C_y(j) < 0.$$

Likewise from the definitions of a two-parameter SSD given by Eqs. (7)–(10) and (13) and a one-parameter SSD given by Eqs. (18) and (19) as well as from Fig. 4 and presented examples, it is apparent that in the position of a true local maximum of a two-parameter SSD (i_t, j_t), the following conditions are satisfied:

$$S(i_t,j_t) > 0, \quad X(i_t,j_t) < 0 \quad \text{and} \quad Y(i_t,j_t) < 0. \tag{20}$$

Together with them the following conditions, saying that in the point (i_t, j_t) one-parameter SSDs have local minima in both dimensions, also must be satisfied:

$$X(i_{t-1},j_t) \geq X(i_t,j_t) \leq X(i_{t+1},j_t) < 0 \tag{21}$$

$$Y(i_t,j_{t-1}) \geq Y(i_t,j_t) \leq Y(i_t,j_{t+1}) < 0. \tag{22}$$

Condition (20) is not satisfied in the point of a false local maximum in a two-parameter SSD (i_f, j_f), i.e.

$$S(i_f, j_f) > 0 \quad \text{and} \quad (X(i_f, j_f) \geq 0 \text{ or } Y(i_f, j_f) \geq 0). \tag{23}$$

By application of condition (20) we can unambiguously distinguish between true and false maxima in a two-parameter SSD. Examples of two-parameter SSD spectra from Figs. 7 and 9, after application of condition (20), are presented in Figs. 10 and 11.

Summarizing the algorithm to search for peaks in a two-dimensional spectrum is as follows:

a. Using Eq. (13), we calculate the spectrum of the smoothed second differences in both dimensions (two-parameter SSD) $S(i,j)$.
b. Using Eq. (14), we calculate the spectrum of the standard deviations of the smoothed second differences in both dimensions $F(i,j)$.
c. For $i \in \langle 0, n_1 - 1 \rangle$, $j \in \langle 0, n_2 - 1 \rangle$, where n_1, n_2 are sizes of the spectrum, we search for local maxima in the spectrum of two-parameter SSD – $S(i,j)$.
d. Once we have found a local maximum in the position i_l, j_l, then by using Eqs. (18) and (19), we calculate spectra of one-parameter SSD $X(i_l, j_l)$ and $Y(i_l, j_l)$, respectively. Then by applying condition (20) we decide whether the found local maximum is true or false.
e. If condition (20) is satisfied we test the slice $S(i, j_l), F(i, j_l)$, $i \in \langle 0, n_1 - 1 \rangle$ (slice F in Fig. 3) by Mariscotti's criteria for the shape of the peak in x dimension. Likewise we test the slice $S(i_j, j), F(i_l, j)$, $j \in \langle 0, n_2 - 1 \rangle$ (slice E in Fig. 3) by the same criteria for the shape of the peak in y dimension.

Fig. 10. Two-parameter SSD spectrum from Fig. 7 after application of condition (20).

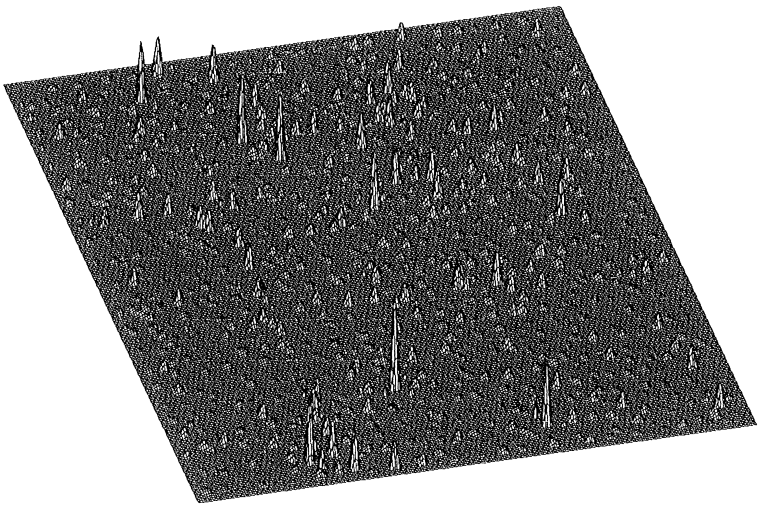

Fig. 11. Two-parameter SSD spectrum from Fig. 9 after application of condition (20).

f. If the shape of the peak in both dimensions satisfies these criteria, we have found a two-dimensional peak in the position i_l, j_l.
g. We repeat the whole procedure for another local maximum from point c onwards.

3. Peak-searching algorithm for three and m-dimensional spectra

The algorithm of peak searching in two-dimensional spectra described so far can be generalized to three-, four- and m-dimensional spectra. Analogous to the two-dimensional case, we can define for three dimensions, a spectrum of the smoothed second differences in all three dimensions, three-parameter SSD,

$$S(i,j,k) = \sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} \sum_{k_1=k-r}^{k+r} C(i_1 - i, j_1 - j, k_1 - k) N(i_1, j_1, k_1) \tag{24}$$

and its standard deviation

$$F(i,j,k) = \sqrt{\sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} \sum_{k=k-r}^{k+r} C^2(i_1 - i, j_1 - j, k_1 - k) N(i_1, j_1, k_1)}. \tag{25}$$

If we choose the filter according to Eq. (10), we obtain

$$C_s(i,j,k) = C(i)C(j)C(k). \tag{26}$$

With a view to explaining some facts in Table 2, we present this three-dimensional filter for $z = w = 0$.

With regard to Eq. (10) from Table 2 we can see that of the expected position of a three-dimensional peak $i_1 = i,\ j_1 = j,\ k_1 = k$ we have

$$C_s(i,j,k) < 0 \tag{27}$$

and

$$C(i) < 0, \quad C(j) < 0, \quad C(k) < 0. \tag{28}$$

Obviously the point $i_1 = i,\ j_1 = j,\ k_1 = k$ is the only one in Table 2 that satisfies conditions (27) and (28). Further in analogy to Eqs. (18) and (19), we define spectra of the one-parameter SSD in all three dimensions by

$$X(i,j,k) = \sum_{i_1=i-r}^{i+r} C(i_1 - i) N(i_1, j, k) \tag{29}$$

$$Y(i,j,k) = \sum_{j_1=j-r}^{j+r} C(j_1 - j) N(i, j_1, k) \tag{30}$$

$$Z(i,j,k) = \sum_{k_1=k-r}^{k+r} C(k_1 - k) N(i, j, k_1). \tag{31}$$

The spectrum of a three-parameter SSD defined according to Eq. (24) is insensitive to continuous background, one-fold coincidences peak–background, two-fold coincidences peak–background or to any combinations of their intersections. Now in the position of the expected three-dimensional peak, the function $S(i,j,k)$ has corresponding local minima. Besides in the vicinity of a three-dimensional peak the function $S(i,j,k)$ contains additional 12 local minima. Again the true local minimum must be distinguished from the

Table 2
An example of three-dimensional filter for $z = w = 0$

	$k_1 = k-1$			$k_1 = k$			$k_1 = k+1$		
i_1 / j_1	$i-1$	i	$i+1$	$i-1$	i	$i+1$	$i-1$	i	$i+1$
$j-1$	1	−2	1	−2	4	−2	1	−2	1
j	−2	4	−2	4	−8	4	−2	4	−2
$j+1$	1	−2	1	−2	4	−2	1	−2	1

false ones. Analogously with condition (20) and in line with the example given above of filter C, one can conclude that in the position of true local minimum (i_t, j_t, k_t) it holds

$$\begin{aligned} &S(i_t,j_t,k_t) < 0, \qquad X(i_t,j_t,k_t) < 0, \\ &Y(i_t,j_t,k_t) < 0, \qquad Z(i_t,j_t,k_t) < 0. \end{aligned} \tag{32}$$

Now we can generalize the search algorithm for m-dimensional spectra. For the spectrum of the m-parameter SSD we can write

$$S(i_1,i_2,\ldots,i_m) = \sum_{a_1=i_1-r}^{i_1+r} \sum_{a_2=i_2-r}^{i_2+r} \cdots \sum_{a_m=i_m-r}^{i_m+r} C(a_1-i_1, a_2-i_2, \ldots, a_m-i_m) N(a_1,a_2,\ldots,a_m). \tag{33}$$

Its standard deviation is

$$F(i_1,i_2,\ldots,i_m) = \sqrt{\sum_{a_1=i_1-r}^{i_1+r} \sum_{a_2=i_2-r}^{i_2+r} \cdots \sum_{a=i_m-r}^{i_m+r} C^2(a_1-i_1, a_2-i_2, \ldots, a_m-i_m) N(a_1,a_2,\ldots,a_m)}. \tag{34}$$

Similarly, the spectra of a one-parameter SSD are

$$\begin{aligned} X_1(i_1,i_2,\ldots,i_m) &= \sum_{a_1=i_1-r}^{i_1+r} C(a_1-i_1) N(a_1,i_2,\ldots,i_m) \\ X_2(i_1,i_2,\ldots,i_m) &= \sum_{a_2=i_2-r}^{i_2+r} C(a_2-i_2) N(i_1,a_2,\ldots,i_m) \\ &\vdots \\ X_m(i_1,i_2,\ldots,i_m) &= \sum_{a_m=i_m-r}^{i_m+r} C(a_m-i_m) N(i_1,i_2,\ldots,a_m). \end{aligned} \tag{35}$$

Provided we have defined filter C, either according to system (9), (10) or to Eq. (12), then for an m-dimensional peak $(i_1, i_2, \ldots, i_m)$ one can conclude that

$$\begin{aligned} &\text{a. if } m \text{ is odd}, \quad S(i_1,i_2,\ldots,i_m) < 0 \\ &\text{b. if } m \text{ is even}, \quad S(i_1,i_2,\ldots,i_m) > 0. \end{aligned} \tag{36}$$

The spectrum of an m-parameter SSD (33) is insensitive to lower-fold coincidences of the peak-background. It is selective only to m-fold coincidence peaks. However, around the m-dimensional peak it generates false

local extremes, if m is odd – minima, if m is even – maxima. Their number is

a. if m is odd, $\dfrac{3^m-1}{2}-1$

b. if m is even, $\dfrac{3^m-1}{2}$.

In analogy to two- and three-dimensional spectra, the false local extremes can be suppressed by applying conditions

$$X_j(i_1,i_2,\ldots,i_m)<0, \quad j\in\langle 1,m\rangle \tag{37}$$

where $X_j(i_1,i_2,\ldots,i_m)$ are calculated according to Eq. (35).

Finally, in the general case of m-dimensional spectra the search algorithm of m-fold coincidence peaks can be expressed as follows:

a. Using Eq. (33), we calculate the spectrum of the smoothed second differences in all dimensions (m-parameter SSD) $S(i_1,i_2,\ldots,i_m)$.
b. Using Eq. (34), we calculate the spectrum of standard deviation of the smoothed second differences in all dimensions $F(i_1,i_2,\ldots,i_m)$.
c. For $i_j\in\langle 0,n_{j-1}\rangle$, $j\in\langle 1,m\rangle$, where $n_1,n_2,\ldots,n_m$ are sizes of the spectrum, we search for local extremes

1. if m is odd – local minima
2. if m is even – local maxima

in $S(i_1,i_2,\ldots,i_m)$.

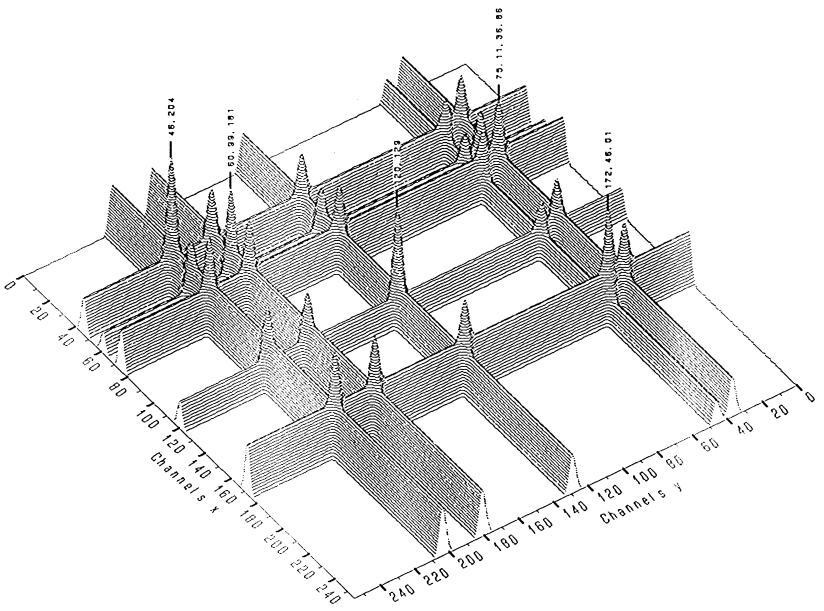

Fig. 12. Synthetic two-dimensional spectrum with found peaks denoted by markers.

d. Once we have found an appropriate local extreme in the point $l_1, l_2, \ldots, l_m$, using Eq. (35), we calculate spectra of one-parameter SSD

$$X_1(l_1, l_2, \ldots, l_m), \ldots, X_m(l_1, l_2, \ldots, l_m).$$

Then by applying conditions (37) we decide whether the obtained local extreme is true or false.

e. If conditions (37) are satisfied, we test the slices

$$\begin{aligned}
&(-1)^{m+1} S(i_1, l_2, \ldots, l_m), \quad F(i_1, l_2, \ldots, l_m), \quad i_1 \in \langle 0, n_1 \rangle \\
&(-1)^{m+1} S(l_1, i_2, \ldots, l_m), \quad F(l_1, i_2, \ldots, l_m), \quad i_2 \in \langle 0, n_2 \rangle \\
&\vdots \\
&(-1)^{m+1} S(l_1, l_2, \ldots, i_m), \quad F(l_1, l_2, \ldots, i_m), \quad i_m \in \langle 0, n_m \rangle
\end{aligned}$$

by Mariscotti's criteria for the shape of the peak in each dimension.

f. If the shape of the peak in all dimensions satisfies these criteria, we have found the m-dimensional peak in the position $(l_1, l_2, \ldots, l_m)$.

g. We repeat the whole procedure for another local extreme from point c onwards.

4. Results

The algorithms to search for peaks in coincidence two- and three-dimensional spectra described so far have been tested using computer-generated synthetic spectra. For this kind of spectra we know in advance

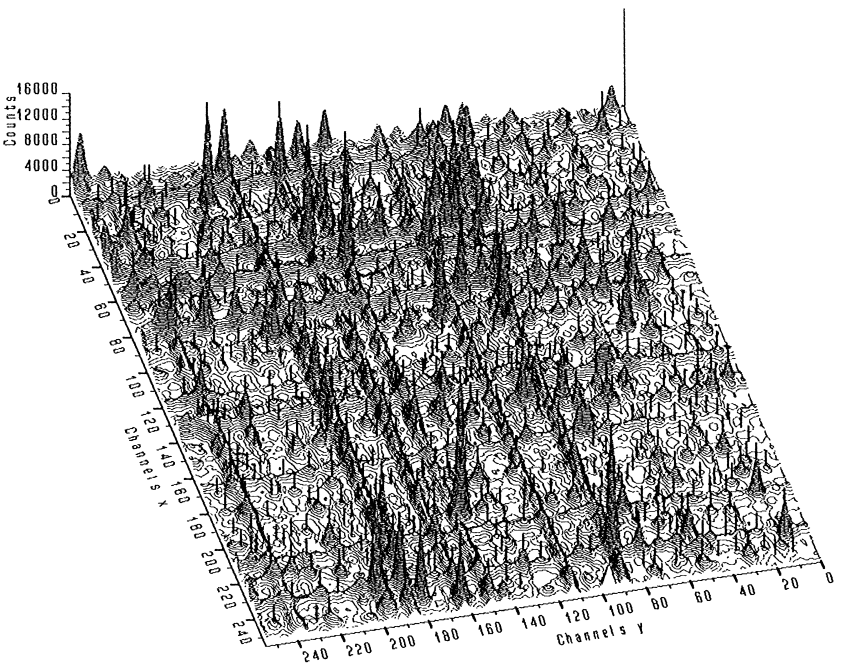

Fig. 13. Coincidence of two-dimensional γ-ray spectrum with found peaks.

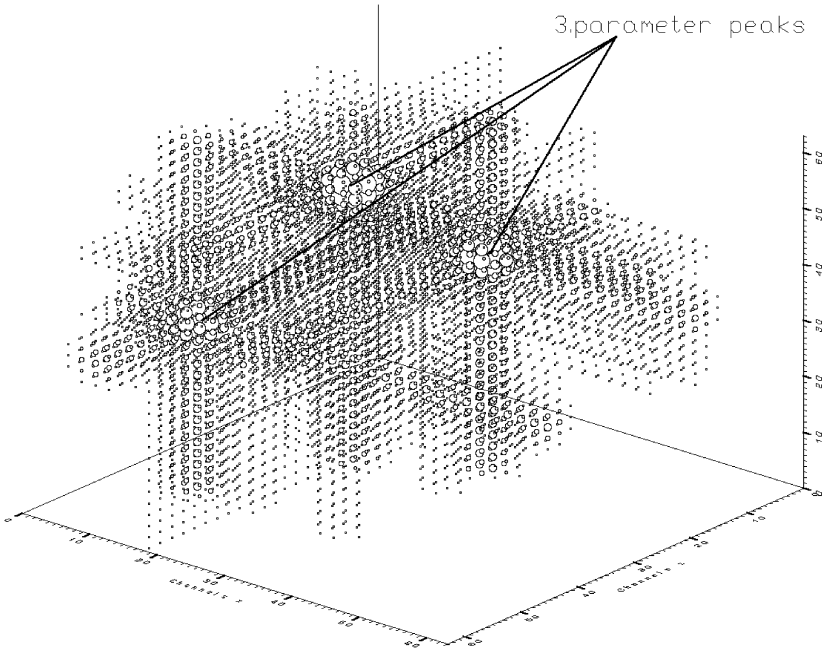

Fig. 14. Synthetic three-dimensional spectrum.

the result that should be obtained and thus we can verify the reliability of the algorithms. In Fig. 12 we can see an example of such a two-dimensional spectrum. It contains two-dimensional peaks, ridges in both dimensions, and noise. The peak-searching procedure finds two-dimensional peaks exactly at the location where they were generated. The markers denote the positions of the found two-dimensional peaks. The method is insensitive to the crossing of ridges.

Fig. 13 illustrates real coincidence two-dimensional γ-ray spectrum. Again, the found two-dimensional peaks are denoted by markers.

Similarly we have tested the peak-searching algorithm for three-dimensional spectra. In Fig. 14 we can see again the synthetic three-dimensional spectrum. The channels are represented by balls. Their sizes are proportional to the counts that the channels contain. The spectrum is composed of three-dimensional peaks, ridges of one- and two-fold coincidences and noise.

Again, the peak-searching procedure determines exactly the positions of three-dimensional peaks in the locations where they were generated. In Fig. 15 we can see the found peaks without spectrum. The employed visualization technique allows us to read out in a simple way the positions of found peaks.

Likewise, we have used the derived algorithm to search for peaks in the real three-dimensional spectrum (Fig. 16).

The peaks found in this spectrum are shown in Fig. 17. Again this example gives evidence that the method is insensitive to crossing of lower-fold coincidences.

5. Conclusions

The paper presents a peak-searching algorithm for two- and three-dimensional spectra. The algorithm was generalized for *m*-dimensional spectra as well. The derived algorithms are insensitive to both lower-fold coincidences, peak–background and their crossings.

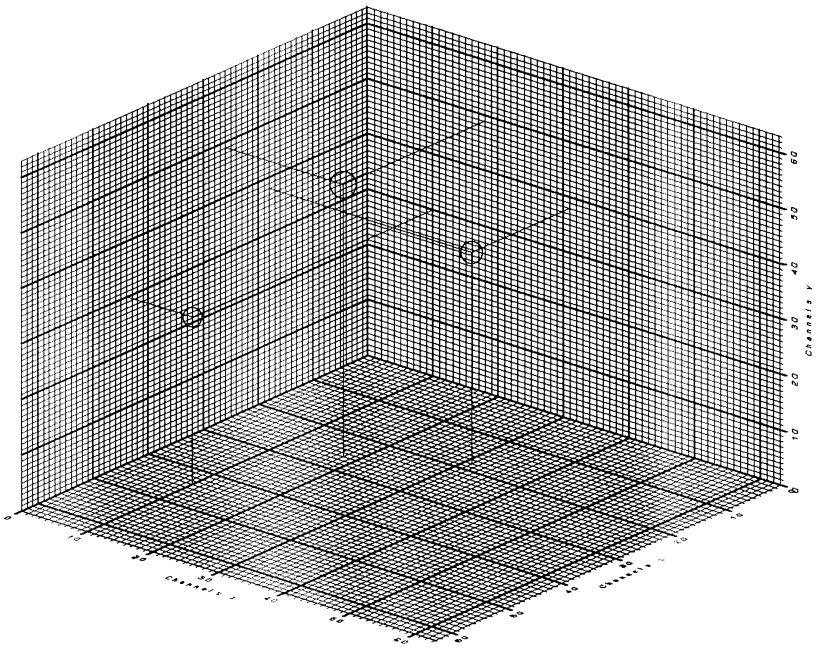

Fig. 15. Found peaks in three-dimensional spectrum.

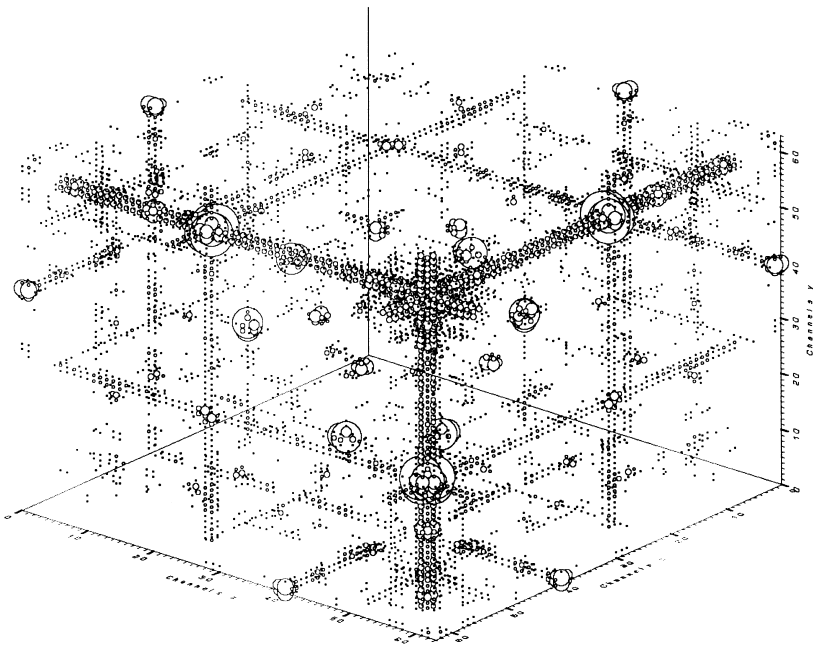

Fig. 16. Real three-dimensional spectrum.

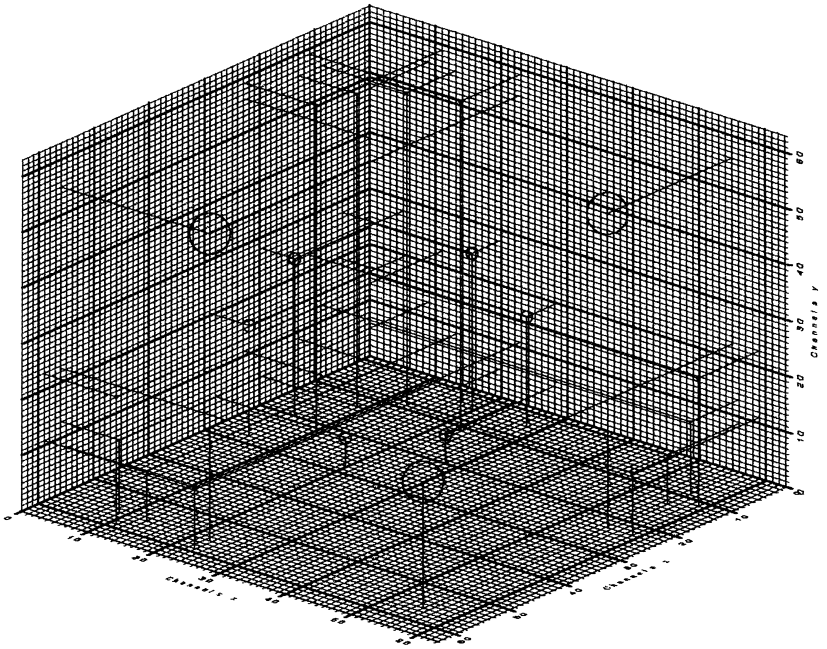

Fig. 17. Found peaks in three-dimensional spectrum from Fig. 16.

The algorithms of peak searching derived in the paper have been implemented in the multiparameter nuclear data acquisition and processing system [11,12]. They have been employed in order to process one-, two- and three-dimensional spectra from experiments in LBL Berkeley, which are carried out in collaboration with Vanderbilt University, JINR Dubna, Idaho National Engineering Laboratory and Oak Ridge National Laboratory.

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